"ALMOST EVERY PROBLEM HE TOUCHED EVENTUALLY TURNED INTO GOLD"

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The title is a quote from my "Yang’s Pyramid" — published in the festschrift celebrating Professor Yang’s 70th birthday, «Chen Ning Yang, a Great Physicist of the Twentieth Century». I likened the collection of his scientific works to a pyramid. As time goes on, hidden treasures continue to be discovered, adding to the well-known brilliant ones. This observation of mine continues to bear out in an impressive way. For the celebration of his 85th birthday, I will highlight recent developments of such remarkable achievements and make an attempt to identify the reasons for them, with the backdrop of my own perspectives.

1. Introduction

On this happy occasion of celebrating the 85th birthday of Professor Yang, I would like to wish him, and us, many, many more.¹

The title is a quote from my "Yang’s Pyramid" — published in the festschrift celebrating Professor Yang’s 70th birthday, «Chen Ning Yang, a Great Physicist of the Twentieth Century», 47–48. Many recent new developments continue to confirm this observation. Also, happily I found that Professor Yang likes it. Among all that has been written about Professor Yang’s accomplishments, this quote from my two-page essay was the one used by Professor John Toll, the then President of Washington College, on the occasion of Professor Yang being honored with the Honorary Degree in Science, see Fig. 1, and printed in their Magazine, Summer 1999 issue. I feel like having hit a jackpot.

¹The day before this conference, 10/30/2007, at the check-in counter of the conference hotel, Swissotel Merchant Court, Singapore, I saw Lay-Nam Chang and his wife. I greeted them, "Great to see you! Long time no see." He said, "Yah, a long time, it must be more than 10 years." I replied, "Weren’t you at Yang 80 in Beijin?" He said, "You are right, I was there." So I said, "Well, it was 5 years since we had seen each other." We all laughed and agreed that we would be lost without the get-togethers for Professor Yang’s birthday celebrations.
It is my extraordinary good luck and privilege to know and to be mentored by a great physicist like Professor Yang. During 1969–1986 I was fortunate to be close to his institute, now the C.N. Yang Institute of Theoretical Physics, SUNY Stony Brook. I learned a lot from him through osmosis around his institute and from direct contacts. He was friendly and generous with discussions. I have three copies of his *Selected Papers, 1945–1980, with Commentary*, one at my office and two at home — one of which is the original hardbound copy with his handwritten comments, Fig. 2. These comments reveal more about his friendliness than the worth of my "valuable advice and criticism".

Professor Yang also initiated my life-transforming move from pure research institution experience at the Institute for Advanced Study (1967–1969) and Brookhaven National Laboratory (1969–1986), to university experience at UC Davis. (On the occasion of the 75th Anniversary in 1983, UC Davis invited Professor Yang to visit and asked him for a list of people to consider for hiring in the Physics Department. He suggested two names, of which mine was one. Thanks to my colleagues at Davis, they chose to go after me and obtained thirteen reference letters for my hiring.) My life has been enriched by the two experiences, in an ideal order in spacetime.

I hope that what I write here is helpful to those who would like to learn a successful way of doing physics from Professor Yang. I thank the organizers for this enjoyable and intellectually stimulating conference.

2007 is the 50th anniversary of many important events. It is the 50th anniversary of Professor Yang’s Nobel Prize with Professor Lee, and of the publications of several of his important papers, some of which I will discuss. 2007 is also the 50th anniversary of the Bardeen-Cooper-Schrieffer (BCS) paper for superconductivity, of the launching of Sputnik by the Soviet Union, of the Broadway premier production of Bernstein’s West Side Story, etc. On a much more minute scale, 2007 is also the 50th anniversary of my graduation from the Tainan Girls School, which I had completely forgotten till my classmates organized our reunion. It was at the Tainan Girls School where our physics teacher, Mr. Run-Chun Li, got me hooked on physics. I still remember how happy I was when I thought that I understood the centrifugal force. 2007 also marks the first year of my new life as a professor emerita. Coming to this conference and giving this talk gave me a valuable opportunity to reflect and to plan ahead as a physicist. I hope that you all will forgive my indulgence in chatting more than probably I should.

2007 happens also to be the year that the Berkeley Center for Theoretical Physics (BCTP) held its official opening symposium. The funding of it was initiated by the donation from Jane and Robert Wilson in memory of their friendship with Oppenheimer. Many interesting talks were given. It is particularly heart-warming to see that the photos of my other two mentors, Professor Chew (my Ph.D. thesis adviser) and Professor Mandelstam, put together with that of Oppenheimer in a small gallery at BCTP. My deep gratitude to Chew and to Mandelstam will be written about in the future.
2. Reasons for "Almost Every Problem He Touched Eventually Turned Into Gold"

Based upon what I know about his papers and him as a person, I would give two main reasons for why "Almost Every Problem He Touched Eventually Turned Into Gold": one, insightful choice of problems to work on; two, thoroughness in the investigation and getting to the primordial sources. The former comes from in-depth understanding of physics and keeping alert about current experiments. The latter requires dedication and perseverance. Both involve being gifted and making best use of it. Some of his papers were recognized quickly. Some were ahead of their times; however, they will eventually shine when their relations to experiments are made. I will illustrate these qualities of his papers next.

3. Examples of Yang’s Papers that Continue to Shine or Have Newly Turned into Gold

Here I list examples of Yang’s papers in the three major fields of the physical sciences. Any one of these papers could be a proud accomplishment of a whole career.

3.1. Particle physics


The paper established local gauge symmetry as principle for interactions in the quantum description. (Yang as a graduate student at Chicago already was thinking about big problems and asked the question of how gauge transform was related to interactions. See Attachment 1.) The demand of covariance of equations of motion under a local phase factors multiplication—called local gauge symmetry—led Yang and Mills derive the equations that now bear their name. This expanded Einstein’s using general coordinate transformation symmetry to derive equations in physics.

A primordial source or truth—e.g., the Pythagorean theorem, Newton’s universal law of gravity, the Heisenberg algebra, the Yang-Baxter relation, or a symmetry law—can be present in many problems, simple or complex. Being able to get to the primordial source of a well chosen problem (relevant and solvable in a reasonable time) is of crucial importance. This quality is pervasive in Yang’s papers.

Pedagogical expositions of these will be given in the two books that I have been writing: "Basic Mathematics for the Physical Sciences" and "Group Theory for the Fundamental Laws of Physics", based upon the material I developed for my teaching.
It took more than twenty years before the true important nature of this paper was realized in physics. After contributions by many distinguished physicists, the theory now has been fully quantized and developed to become the theory for electroweak and strong interactions, called the Standard Model. See the book, "50 Years of Yang-Mills Theory," edited by G. 't Hooft. With the verifications in experiments/observations, it has been established that the local gauge symmetry and Einstein's general coordinate transformation symmetry form the bases (primordial sources) of all major interactions: electroweak, strong, and gravity. These physics developments also have a profound influence on mathematics. (How to quantize gravity theory fully is still under intense research.)

The formal citation number of the paper is about 1450, which averages about 27/yr. It peaked to about 80/year during the late '70s to early '80s and now stays about 20/year. There are probably as many, if not more, informal citations using the term Yang-Mills without citing the paper.\footnote{The citation data given in this section were as of March, 2008.}


Responding to the so called $\theta$-$\tau$ puzzle from weak decay experiments, the paper questioned the validity of the parity symmetry (a global symmetry), $P$, and stimulated experimenters to look into it. Within a year of its publication, experiments confirmed the violation of $P$. In about fourteen months after the publication of their paper, Lee and Yang were awarded the Nobel Prize. This paper is among the top few that has had the shortest time between the publication and the award of a Nobel Prize to its authors.

Currently this paper has about 1000 citations, averaging about 20/year and in recent years staying around 20–30/year.


The paper raised questions about the violation of other global symmetries, $C$ (charge conjugation) and $T$ (time reversal). Although this paper missed the discussion of $CP$ violation, it set up the parametrizations which are useful for the now called the mass-matrix $CP$ violation.

Currently this paper has about 300 citations, averaging about 6/year and staying steadily at that.

Wu-Yang 1964, Phys. Rev. Lett. 13, 380–385, "Phenomenological Analysis of Violations of $CP$ Invariance in Decays of $K^0$ and $\bar{K}^0$"
Responding quickly to the observation of \( CP \) violation in 1964, this paper analyzed the \( CP \) violation and became the standard reference for the analysis of the now called mass-matrix \( CP \) violations.

Later, the Kobayashi-Maskawa framework for \( CP \) violation allows direct (or amplitude) \( CP \) violations as well as mass-matrix \( CP \) violations. So this Wu-Yang paper is still relevant. My recent paper showed, among other results, how the Wu-Yang phase convention is always possible in general.\(^5\)

Currently this paper has about 259 formal citations, averaging about 6/year and staying about a few per year. There are many citations of Wu-Yang phase convention without formally citing the paper.

Eventually global symmetries of \( P \), \( CP \), and \( T \) have all been established to be violated in weak interactions. However, these violations observed in the laboratories are found not sufficient to explain the lack of anti-matter in the Universe. Whatever the future resolution will be, the generic analysis made in these papers by Yang and coworkers will be relevant.


This paper introduced and emphasized the loop formulation.

The loop formulation has been widely used; however, it is commonly called the Wilson-loop. Based upon my research on the contents and the times of the publications of this paper and those of the paper by K. Wil-

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\(^5\)My recent paper, L.-L. Chau, "Phase Direct \( CP \) Violations and General Mixing Matrices", Phys. Lett. B 651 (2007) 293–298, was prompted by the beautiful experimental work of the Belle collaboration. A. Garmash et al., "Evidence for Large Direct \( CP \) Violation in \( B^\pm \to \rho^0 K^\pm \) from Analysis of Three-Body Charmless \( B^\pm \to K^{\pm \pi^\pm \pi^\mp} \) Decays", Phys. Rev. Lett. 96, 251803 (2006). It was the first experiment of its kind. It confirmed what I wrote and advocated in the 1980s: an observation of \( CP \) violation in charged \( B^\pm \), rather than \( B^0 \), would give the cleanest evidence for the direct \( CP \) violation in the amplitude; and the reaction \( B^\pm \to \rho^0 K^\pm \) had been the number one on my list; see my overview paper, "In Search of \( CP \) Noninvariance in Heavy Quark Systems", 249–268 of \( \leq CP \) Violation\(\rangle \) edited by C. Jarlskog, 1989, World Scientific.

Here I highlight the new results in this paper: formulating expressions for amplitudes suitable for quantifying both modulus and phase direct \( CP \) violations and discovering Möbius transformation (MT) relations, which provide encouraging information for the search of direct \( CP \) violations in general; then applying the formulation and calculating the measurements of phase direct \( CP \) violations and strong amplitudes by the Belle collaboration; showing a versatile construction procedure for \( N \times N \) Cabibbo-Kobayashi-Maskawa (CKM) matrices, Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices, and general unitary matrices, thus giving a full derivation of the parametrization for the \( N \times N \) cases and a comprehensive understanding of the \( 3 \times 3 \) case, which was originally constructed through trials by Chau-Keung, Phys. Rev. Lett. 53 (1984) 1802–1806 and has since been used by the Particle Data Group as the standard parametrization.
son, "Confinement of Quarks", Phys. Rev. D 10 (1974) 2445–2459, I have formed the opinion that the loop ought to be called the Yang-Wilson-loop. In celebrating Professor Yang’s 80th birthday, I wrote it up in the article, "A Forgotten Treasure in Yang’s Pyramid", which was distributed but unpublished. I attach it here as Attachment 2, so you can see the details and an interesting encounter with E.P. Wigner.

Currently this paper has about 350 formal citations, averaging about 10/year and staying about a few per year. (I am sure that the citations would have been more if people understand what I have said about it.)


This paper extended the previous one, and in particular included the presence of the Dirac magnetic monopole.

Currently this paper has about 800 citations (some overlapping with Yang 1974), averaging about 25/year and recently staying about 20/year.

**For additional important papers by Yang in particle physics**, see the talk by T.T. Wu at this conference.

### 3.2. Many-body physics (which includes statistical mechanics and condensed matter physics)


These two papers clarified the mathematical description of liquid-vapor phase transition and discovered the famous Lee-Yang circle theorem.

Paper I has about 1000 citations, averaging about 18/year. Impressively, in recent years its citations have stayed about 30/year. Paper II has about 1200 citations (with some overlap between I and II), so averaging about 22/year. Recent citations stay about 30/year, above the average.


Impressively, the beyond mean-field-theory-approximation results of this paper have been confirmed on the BEC side of the BCS-BEC crossover experiment by A. Almeyer et al., Phys. Rev. Lett. 98, 040401 (2008).

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6BEC stands for Bose-Einstein condensation. For BCS, see Footnote b. For information
Currently this paper has about 430 citations, averaging about 9/year and in recent years staying about 15/year, above the average. In 2004, the number of citations surged to 25.

For other aspects of this paper, see K. Huang’s talk, in the forthcoming Proceedings or Int. J. Mod. Phys. B 21, 5059–5073 (2007).

Yang 1962, Rev. Mod. Phys. 34, 694–704, "Concept of Off-Diagonal Long-Range Order and the Quantum Phase of Liquid He and of Superconductors"

The paper gave a comprehensive and exact mathematical description of superfluidity/superconductivity. It coined the now standard term ODLRO (off-diagonal-long-range-order).

Currently this paper has about 730 citations, averaging about 16/year and in recent years staying about 25/year, above the average. In 2007 the number of citations surged to 30.


This paper discovered the now called Yang-Baxter relation and Yangian. With my coworkers, I had derived generalized Yang-Baxter relations in theories in higher dimensions. I pointed out that the Yang-Baxter relations to the quantum exchange algebras are as basic as Jacobi identities are for algebras, thus as ubiquitous,\(^h\) Not surprisingly, there is a recent surge of interest in this topic related to the string theory.

Currently this paper has about 1020 citations, averaging about 26/year and in recent years staying over 30/year, above the average. In 2007, the number of citations surged to 43. There are many citations of Yang-Baxter or Yangian without formally citing the paper.

For other information about this paper, see the talk by M.L. Ge.


Currently this paper has about 600 citations, averaging about 18/year and in recent years staying about 25/year, above the average. In 2007, the number of citations surged to 28.

For additional important papers by Yang in many-body physics, see the talk by F.Y. Wu at this conference.

3.3. Astrophysics/cosmology

Yang did not have any paper directly in this field. However, because astrophysics/cosmology uses particle physics and many-body physics together with Einstein’s general relativity, it is impacted by Yang’s papers.

4. Outlook

The 20th Century had seen fantastic progress in the physical sciences: establishment of quantum phenomena and description; rational understanding of many-body physics and observations of many striking phenomena; development of the Standard Model of electroweak and strong interactions and its experimental confirmation; profound advancement of our understanding of spacetime and the observation of the expansion of the Universe and the imprint of the Universe at around 300,000 years after the Big Bang which occurred about 14 billion years ago; etc.

The 21st Century began with continued confirmations of the striking observation made toward the end of the 20th century: the matter that the stars and you and I are made of constitutes only about 5% of the mass/energy of the Universe, while dark matter takes up 20% and dark energy takes up 75% and is responsible for the acceleration of the expanding Universe! We are facing many exciting challenges.

What is quantum gravity?
How do particles get their masses?
How does the lack of anti-matter in the universe come about?
What is the microscopic description for high Tc superconductivity?
What is dark matter?
What is dark energy?
Whatever the future developments will be, Yang’s papers will continue to be relevant and contributing because they have brought about discoveries of the foundations in physics.

Nature, in Her precise language of mathematics, constructs interactions according to symmetry. However, symmetry does not dictate interaction uniquely. Interestingly, Nature always chooses the simplest possibility allowed by symmetry, e.g., the actions for electroweak, strong, and gravity are the simplest possible ones; the groups established in the Standard Model are the simplest Lie groups, $U(1) \otimes SU(2) \otimes SU(3)$. Nature uses symmetry breaking to make our world and the Universe more interesting: e.g., the violation of the global symmetries of $P$, $CP$, $T$ in weak interactions; the violations of local gauge symmetries in the phenomena of superfluidity/superconductivity and in the mass generations in the Standard Model. I would like to show my favorite artist’s painting which I consider to have wonderfully captured Nature’s use of symmetry and symmetry breaking to make our world beautiful and interesting, Fig. 3.

To describe the situation, I modify a quote from my Nobel Laureate classmate, David Gross (mentored by Geoffrey Chew as thesis adviser), in celebrating Yang 70, "Symmetry dictates interactions and Yang dictates symmetry", to "Symmetry plus simplicity dictates interactions, Einstein and Yang dictate symmetry, but who/what dictate symmetry breaking?" A (permuted) quotation of Einstein says it well, "Malicious the Lord is not, but subtle He is!"

We eagerly want to find out more about how exactly Nature does all these. I like to hold the philosophical view that all are out there in Nature: mathematics, music, etc., as well as whatever the human brain can come up with. I concocted a picture for this point of view, Fig. 4. This point of view resonates with what Michelangelo had said about his marble sculpturing (which I am paraphrasing), "The figures are already in the marble, all I have to do is to chip away the unnecessary." Let us follow the role models of Einstein and Yang, be all each of us can be, chip it away and find out more of Nature’s beautiful simplicities, primordial sources, and subtleties.

**Acknowledgements**

I would like to give my heart-felt thanks to Karen Andrews, William and Gloria Rogers, and Weiben Wang (my son) for reading the manuscript and giving valuable and encouraging comments.
Attachment 1:

Transcript of Notes by C.N. Yang,
Written in March 1947 at University of Chicago

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Chau’s note: I made the transcript of the Notes for the convenience of the reader. I referred to it when I discussed the 1954 Yang-Mills paper in my talk at the Conference celebrating Professor Yang’s 85th birthday in Singapore. I thank Professor Yang for proof reading the transcript and acknowledging that he did not find any mistakes in the transcript given here, March 2008.

The photocopy of these Notes was published in “50 Years of Yang-Mills Theory”, edited by G. ‘t Hooft. In the introduction to the photo copy of these notes, Professor Yang gave the following references:

[1] W. Pauli, Handbuch der Physik 24 (1933),

Gauge Invariance and Interaction

I. The gauge invariance in electromagnetic theory serves for two purposes:

(i) Give rise to the definition of a charge and current density,
(ii) Fix the interaction between an arbitrary field and the photon field by the requirement of the gauge invariance.

By the second purpose, e.g. a real field cannot interact with the photon. (But notice that a complex field that does not undergo a change under a gauge transformation will not interact with the photon either, e.g. the neutron field belongs to this class.)

II. We can easily formulate a theory for meson gauge transformations by requiring that $U_k \rightarrow U_k e^{i\alpha_k}, U_k^* \rightarrow U_k^* e^{-i\alpha_k}$ and get a "meson charge and current" density, where $\alpha$ is a constant, ($\varepsilon_k$ = meson charge number of the particle $U_k$).

In order to fix the interaction between an arbitrary field & the meson field a gauge transformation of the second kind for the meson field is
necessary. This can be done in the case of neutral mesons by writing

\[ \mathcal{L} = -\frac{1}{2} F_{\mu \nu} F^{\mu \nu} - K^2 U_\nu U_\nu + 2 A_\nu M_\nu - 2 \frac{1}{K} B M_{\nu \nu} \]

where \( A_\nu \) is Stuckelberg’s potential (c.f. Pauli, R.M.P.) & regarding \( A_\nu \) & \( B \) as independent variables. Gauge transformation:

\[ A_\nu \rightarrow A_\nu + \frac{1}{K} f_{\nu \nu} \]
\[ B \rightarrow B - K f \]

For charged mesons, the theory perhaps works too, with \( \alpha = \) an operator not commutable with \( \tau_3 \) (isotopic spin).

III. Notice that for photon field, \( \varepsilon_k = 0 \) or 1 because all particles are either uncharged or have charge \( e \) in photon field. But for meson field

\[ \frac{G}{e} = \varepsilon_k \approx 3 \text{ if } U_k = \text{ heavy particle field,} \]
\[ \frac{g}{e} = \varepsilon_k \approx 10^{-8} \text{ if } U_k = \text{ light particle field.} \]

IV. The gauge transformation described in II will not lead to any fixation of the interaction because the field part of the \( \mathcal{L} \) and the particle part are respectively invariant under the transformation, while in the e.m. case they both vary.

This constitutes perhaps a fundamental difference between a field with \( K = 0 \) & with \( K \neq 0 \).

**Masses of Particles**

V. The present theory cannot give a satisfactory account of the masses of the different particles because of infinite difficulties and incompleteness of the theory, e.g. we cannot expect to be able to derive the mass ratio of the proton and the electron, of the proton to the neutron in the present theory.

In a completely satisfactory theory there should be only three universal constants: \( \hbar, c, G \) (gravitational constant) which in natural units are all 1. The mass or Compton wave lengths of the particles can be derived.

We see that a theory of such kind is necessarily a merge-together of the Q. theory & general relativity (in which the masses of the particles are defined).

Point worth noticing: if electron radius = range of meson forces, \( M_{\text{meson}} = 137 \text{ m.} \)
Gauge Invariance, Charge Density & Neutral Particles

* If we separate each complex wave function into its real and imaginary parts,

\[ U = v + iw, \]

the gauge transformation becomes

\[ v \rightarrow v + \alpha w, \]
\[ w \rightarrow w - \alpha v. \]

\[ \frac{\delta \mathcal{L}}{\delta \alpha} = 0 \]

represents gauge invariance & leads to a current \( S_\mu \) with divergence = 0,

\[ S_\mu = \text{const.} \left[ \frac{\partial \mathcal{L}}{\partial v^K} w - \frac{\partial \mathcal{L}}{\partial w^K} v \right]. \]

* For particles of spin 0 & 1, \( \mathcal{L} \) has terms \( v \cdot v \), \( w \cdot w \) but no \( vw \).

\[ \therefore S_\mu = \text{term } vw. \]
\[ \therefore S_\mu = 0 \text{ when } v = 0 \text{ or } w = 0. \text{ which gives Neutral particles} \]

\[ \text{""" } ", " " \text{ " } 1/2, \mathcal{L} \text{ has the term } v \cdot w, \text{ no } v \cdot v, w \cdot w \]

\[ \therefore S_\mu = \text{term } vw \text{ and } ww. \]
\[ \therefore S_\mu \neq 0 \text{ when } v = 0 \text{ or } w = 0. \text{ Thus we has no neutral particle.} \]

If, however, we adapt Heisenberg’s recipe for passing to q-no. theory \( S_\mu \) have \( v \cdot w \) term again & we can have neutral particles. But this seems unsatisfactory because if put \( w = 0 \) from the beginning we don’t get an \( \mathcal{L} \).

(See Pauli p.225, 228)
Attachment 2: This is an article I wrote for the celebration of Professor Yang’s 80th birthday. In it I thanked Professor Yang for the important physics I learned from him, related an interesting encounter with Eugene Wigner, and pointed out that the commonly called Wilson-loop ought to be called Yang-Wilson-loop, based upon my research on the contents and the timing of the publications of their papers. This article was distributed but not published. I attach it here as a reference for my discussion of the Yang 1974 paper in the main text.

A Forgotten Treasure of Yang’s Pyramid

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On the occasion of celebrating Professor Yang’s 80th birthday, I would like to express my deep gratitude to him.

During seventeen years (1969–1986) of working at Brookhaven National Laboratory and living at Setauket, which is closer to Yang’s institute in the State University of New York at Stony Brook than the Lab, I benefited tremendously from Professor Yang and from his institute (which was designated to be “C.N. Yang Institute for Theoretical Physics” since his retirement in 1999). Besides welcoming me to attend seminars and to interact with visitors from all over the world at his institute, he generously allowed me to learn physics directly from him. He would spend hours discussing physics with me in his always superbly organized and elegant office, answering questions and giving guidance. The amazing thing was that he could always find the references, the books or the papers he was looking for. What he provided at his institute helped to broaden and deepen my appreciation and understanding of physics and the related mathematics. Actually, it was not my unique experience. Many other scientists had a similar experience. The environment he provided at his institute was truly unique in the world of frontier physics research. I still do not know how he managed to do it.

An anecdote can demonstrate the importance of what I learned through osmosis from his institute. In 1980 I went to the Erice School as one of the speakers. When I first arrived and went to the top part of the building of the school, which had a beautiful view of the blue Mediterranean Sea,
for some reason only Professor Wigner was there. He recognized me from my Princeton days at the Institute of Advanced Study (1967–1969). We started to chat. He asked what I was going to talk about. After hearing the titles of my two scheduled talks, he asked me in his usual overtly polite manner, “Why can’t electromagnetism be completely described by the fields? Why do we need to use the gauge potentials?” The answer to his question seemed to me obvious because of what I had learned from the discussions at Yang’s institute. However, before I gave the answer to Wigner, my mind quickly turned up the story told by Professor Sam B. Treiman at one of the parties at his home in Princeton. Wigner was famous for going around to ask professors at Princeton his questions in his famous super-polite manner. However, people learned that it was dangerous to answer Wigner’s questions. Often, one could easily make a fool of oneself, because Wigner usually had already thought very deeply about his questions. So Treiman said that he never answered Wigner’s question about dispersion relations. He just kept on returning Wigner’s almost-90-degree bows and asked, “What would you like to know about dispersion relations, Professor Wigner?”

Despite my memory of Treiman’s warning, I ventured to give my answer: “Professor Wigner, it is because of the Aharonov-Bohm effect.” [Y. Aharonov and D. Bohm, Phys. Rev. 115 (1959) 485–491.] He abruptly straightened his back and gave me a 90-degree bow, “Thank you, I have had my lesson today.” He then walked away. I did not quite know what to make of it and we never talked about it again during the conference. To my total surprise, in Wigner’s summary talk for the conference he acknowledged me for reminding him of the Aharonov-Bohm effect. Indeed, he had thought more about the problem than I had. When I returned to Brookhaven, I mentioned the story to Dr. Maurice Goldhaber. Maurice said, “Wigner will not acknowledge you in writing.” I did not ask what prompted Maurice to say that. A long, long time afterwards, in 1983, the proceedings of the conference arrived and I just automatically put it on the shelf without looking inside, as I often did with conference proceedings. Quite sometime later, Maurice’s comment reappeared in my mind and I curiously looked up Wigner’s written version of his talk in the proceedings: International School of Subnuclear Physics 1980, Erice, Italy, “The High Energy Limit,” Ed. A. Zichichi, Plenum Press. There it was, Wigner acknowledged me in writing! What Wigner said in his summary talk “What We Have Learned,” 1065–1073 of the Proceedings, about electromagnetism was very insightful. I would like to quote it here:
“Let me recall that when I learned about the description of the electromagnetic field by a gauge quantity, the electromagnetic potential, I did not like it. The gauge-like quantities cannot be measured and have, therefore, no ‘reality.’ Thus, for instance, one can add an arbitrary gradient to the electromagnetic potential without changing any of its effects. This shows that at least in classical theory, it cannot be an observable quantity and this remains true in quantum theory. Since I had an aversion to gauge theories, it was good that Dr. Chau reminded me of the article of Aharonov and Bohm which showed the usefulness of the gauge concept. It showed that if the electromagnetic field is described by a gauge-field, its effect on the charged particle traveling through it is local — if it is described by the electromagnetic field, electric and magnetic, no local description is possible. The interference of two electron beams which originated from a single beam is not uniquely given by the electric and magnetic fields through which they travel — the fields between the two beams also have an effect. However, if the field is described by the electromagnetic potential, the description is local — the potential difference between two points contains implicitly the electromagnetic field between those two points. Thus in quantum mechanics we have to choose between a non-local [effect] by an observable field on the one side, and a local description of the effect by a non-observable field. Experience shows that the second description is much easier and it is reasonable, therefore, to adopt it. This has been done.”

This year, 2002, is Wigner’s 100-year anniversary. On this occasion of the remembrance of Wigner, I am thankful to Wigner for his kindness and for giving me that valuable experience.

As insightful as Wigner was, clearly he did not read or think of what Yang wrote in his 1974 paper, “Integrable Formalism for Gauge Fields,” Phys. Rev. Lett. 33 (1974) 445–447, in which Yang clarified and emphasized what are the essential quantities that describe the local gauge theories — not the fields, not the gauge potentials, they are the non-integrable phase factors, which manifest the Aharonov-Bohm effect in electromagnetism.

In the late 1970’s, through seminars and talks, I learned from Yang’s institute about the Aharonov-Bohm effect and its importance. It became so obvious to me that I could answer Wigner’s question. However, not only did I not learn about the effect in my student days, most graduate students even now do not learn it. Most quantum mechanics and electromagnetism textbooks do not mention the effect. I have taken what I learned from Yang and Yang’s institute by heart and have always taught it when I teach quantum mechanics and mathematical physics. (I have yet to teach a course
on electromagnetism. When I do, I will teach it too.) It is so easy and beautiful for students to understand, and it is so important as Yang has argued and advocated.

As it is well known, the importance of these phase factors was also independently noticed by K.G. Wilson, “Confinement of Quarks,” Phys. Rev. D 10 (1974) 2445–2459. I checked the days of submission and publication of the two papers: Yang’s was submitted on June 10, 1974 and published on August 12, the same year; Wilson’s was submitted on June 12, 1974, and published on October 15, the same year. They were amazingly close. Neither of them knew each other’s paper. (I recently checked with Professor Yang and he confirmed it and indicated that he never had any communications on the topic with Wilson, before or after the publication of the two papers.)

Wilson’s paper has been universally quoted. For some reason Yang’s paper and point of view have not been mentioned when people quote Wilson’s paper. So I have started to mention to people that the “Wilson loop” ought to be called “Yang-Wilson loop,” in that name order because Yang’s paper clearly stated and emphasized that the “loops,” i.e., the non-integrable phase factors, are the defining quantities for local gauge theories; while Wilson’s paper mainly discussed the utilities of the loops.

“Yang’s Pyramid” was the title of my paper contributing to the proceedings of Professor Yang’s 70th birthday celebration: “Chen Ning Yang, A Great Physicist of the Twentieth Century,” Ed. C.S. Liu and S.-T. Yau, International Press. Thus originates the title of this essay.

I have many other things to thank Professor Yang for. They will require some length to write. I think I will save them for the celebrations of his 90th, 100th, ···, birthdays.
Fig. 1. The photo is from the Washington College Magazine, Summer 1999 issue, taken by Jim Graham of Washington College '81 on the occasion of Professor Yang (6th from the right) being honored with the Honorary Degree in Science and Mr. J.F. Kennedy Jr. (2nd from the right) giving the commencement speech. Fourth from the right is Professor John Toll, the then President of Washington College. His presidency (1965-1978) at SUNY, Stony Brook had much to do with Professor Yang’s move in 1965 from Institute for Advanced Study at Princeton to become the Albert Einstein Professor of Physics of SUNY, Stony Brook and the first director of the now C.N. Yang Institute for Theoretical Physics.
Fig. 2. Professor Yang’s handwritten comments on Chau’s copy of his Selected Papers, 1945-1980, With Commentary. English translation of the comments: <Ling-Lie: Thank you for your "valuable advice and criticism". Chen Ning, '83 summer.>
Fig. 3. Georgia O'Keeffe's 1935 painting, "Ram's Head, White Hollybock – Hills", image from the web page of Brooklyn Museum (Bequest of Edith and Milton Lowenthal). I consider it to have wonderfully captured Nature's use of symmetry and symmetry breaking to make our world beautiful and interesting.
Fig. 4. I concocted this picture to convey my philosophical view that all are out there in Nature: mathematics, music, etc., as well as whatever the human brain can come up with. The background image is from the web page "Inside the Eagle Nebula" of T.A. Rector and B.A. Wolpa, NOAO/AURA.