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Stanley Mandelstam, 1928–2016: Brief Biography and Selected Publications with Commentary[1]

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The enduring influences of Stanley Mandelstam's publications are deep and diverse. They affect almost all the current major research efforts in theoretical and mathematical physics that try to deepen our understanding of the physical universe. Reviewing Stanley's accomplishments offers a rare opportunity for everyone interested, experts as well as nonexperts, to gain a perspective about the current status of theoretical and mathematical physics and what to look for in the future. This paper presents a brief biography of Stanley and a selection of his publications, grouped together according to subject matters, with commentary.[2]

Fig. 1. Stanley Mandelstam at the 1961 Solvay Conference.[3]

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[1] A contribution to the "*Memorial Volume for Stanley Mandelstam*," editors N. Berkovits, L. Brink, L. L. Chau, K. K. Phua and C. Thorn (World Scientific Publishing, to be published in 2017). It is referred to as "the *Memorial Volume*" in the rest of the paper.
[2] For a very short highlight of this paper, see author's Physics Today Obituary for Stanley Mandelstam, May issue 2017. About the genesis of these two papers see the Acknowledgments in this paper.
[3] The photo is cropped from the wall-size photo of the 1961 Solvay conference on the 4th floor of the UC Davis Physics Department. (The author had it installed in the late 1980s.)

Contents

1. Brief Biography

Stanley Mandelstam was born in 1928, in Johannesburg, South Africa, and was the elder brother to a sister. His father was a grocer who had recently emigrated from Latvia. His mother was an elementary school teacher, born in South Africa to parents from Latvia.

He obtained a B.Sc. in chemical engineering in 1952 from University of Witwatersrand (or Wits), Johannesburg. By then, he had manifested his talent in mathematical physics with the publishing of the book [A], "*Variational Principles in Dynamics and Quantum Theory*," with Wolfgang Yourgrau.

Subsequently Stanley switched to the study of theoretical and mathematical physics — his true passion, on which he worked for the rest of his live. He obtained a B.A. in two years at Trinity College, Cambridge in 1954. In just another two years he received his Ph.D. from the University of Birmingham in 1956, under the direction of Rudolf E. Peierls, who had brought prominence and high visibility to theoretical physics there. Stanley's thesis work was published in two papers [B] in the *Proceedings of the Royal Society of London A*. By then, Stanley had developed a solid mastery of quantum field theory (QFT), including its use in calculating S-matrices. He would put these to brilliant uses in his work throughout his life. His writings demonstrate his attention to the close relevance between theory and experiments.

After continuing his research at Birmingham for another year, in 1957 he moved to New York City, hired as a Boese fellow, a research position in the Department of Physics at Columbia University 1957–58. He published the 1958 paper [C1], in which he pioneered the double dispersion relations for scattering amplitudes (elements of an S-matrix), now called the Mandelstam representation. It was a daring leap from the insights he had gained from perturbative QFT results and his advanced knowledge about functions of more-than-one complex variables.[4] His presentation of the paper at the 1958 American Physical Society (APS) Washington DC meeting caught the attention of Geoffrey (Geoff) Chew. The two met and had a discussion right after Stanley's talk. At the end of their discuussion, Chew made an offer on the spot to Stanley to go to UC Berkeley as a researcher. Stanley immediately agreed.

[4] A subject of mathematics that is still rarely taught to graduate students in physics.

He had two productive years, 1958–60, at UC Berkeley. He consolidated the Mandelstam representation into its final form using the Mandelstam variables, and published several single-author papers. He worked with Chew on three published papers to implement the Mandelstam representations into the S-matrix approach that Chew had developed earlier with collaborators. He initiated the idea of using Regge poles to treat the high energy behaviors in S-matrices among colleagues at Berkeley.

Then, he was hired back to Birmingham as a professor in 1960. Stanley's research output continued to be spectacular. He wrote an extensive review to summarize his work done at Berkeley, "Dispersion Relations in Strong-Coupling Physics." He published a paper with Chew and Steven Frautschi incorporating the Regge pole idea into the S-Matrix approach. Moreover, he published a paper with Ronald F. Peierls[5] and collaborators at Birmingham, applying the S-matrix approach. Some of these papers are listed in [C1] and [C2]. Stanley further worked by himself not only delving deeper into the complex plane of angular momentum with Regge poles and cuts and publishing many papers, e.g., the 1962 paper in [C5], but also striking out to pioneer two far-reaching new directions in research. He formulated quantum gauge theories in terms of gauge-invariant path-dependent fields (for Maxwell theory in 1962, and then also for Yang–Mills theory in 1968 when he derived the Feynman rules) [C3]. In the same journals, together with the two papers on gauge theories, were his two papers on quantum general relativity: the 1962 one giving the formulation in terms of coordinate-independent path-dependent fields, and the 1968 one deriving the Feynman rules [C4]. The importance of these two breakthroughs are discussed in (S3) and (S4) below. We call these formulations the Mandelstam path-field formulations, for gauge theories and for general relativity.

Thus in 1958–62, he made breakthroughs on four major subjects in theoretical physics, listed and commented upon below in (S1–S4), which correspond to the selected papers [C1–C4], and paved the ground work for the next breakthrough, subject (S5) corresponding to [C5]. These years can be called

"***Stanley Mandelstam's miracle years, 1958–1962***."

Following the 1958–60 publications [C1, C2], Stanley was invited to participate in the prestigious 1961 Solvay conference, celebrating the 50th anniversary of the famous inaugural 1911 Solvay conference on physics,[6] where he gave a talk and published the 1961 paper listed in [C1]. He was one of the youngest participants and appeared in the photo with so many distinguished theoretical physicists. In 1962 he was elected to become a Fellow of the Royal Society, a great honor for any physicist and even more so for a young 6-year post-Ph.D. physicist.

[5] Ronald F. Peierls (1935–2003) was the son of Sir. Rudolf E. Peirels. He later went to Institute for Advanced Study (IAS), Princeton NJ, 1961–62, and then to Brookhaven National Laboratory (BNL), NY, for the rest of his life. The author was a colleague of his and coauthored with him at BNL during 1969–86. He has been dearly missed. See more in the reference at (C2)

[6] M. Curie and A. Einstein attended the 1911 Solvay conference and appeared in that iconic conference photo with other distinguished physicists.

In 1963 Stanley returned to UC Berkeley as a professor in the Department of Physics.[7] He continued to produce ground-breaking work, e.g., those publications after 1963 in [C5–C9]), and make contributions to the subjects (S1–S9) listed below with comments. There were the development of the precursor, in the S-matrix approach culminated in 1968, for the eventual discovery of string theory;[8] the elucidation of mechanisms for quark confinement in quantum chromodynamics (QCD) in 1975-79; the nonperturbative constructions of the bosonization (or fermionization) in (1 + 1)-dimension QFTs in 1975; the proof of the perturbative UV finiteness and $\beta = 0$, in any gauge to all orders, of N = 4 supersymmetric YangMills theory (SYM) in 1983; and from 1968 onward, many important contributions to the development of string theory that culminated in 1992 with the first proof of the pertubative UV finiteness of string theory, so string theory can be considered as a contender for the theory of quantum gravity.

In 1994 he became Professor Emeritus.[9] He continued to do research, use his office in the Department, and live in the same apartment he rented since 1980 in Berkeley (by choice, he was always a renter) until his death on June 11, 2016, age eighty-seven.

With his quiet, always polite, attentive, and kind ways, Stanley won the respect and love from his colleagues and friends. He is deeply missed.[10]

2. Selected Subjects of Publications and Commentary

Stanley made long-lasting major contributions to theoretical physics, which are here organized into the following nine diverse subjects (S1–S9), corresponding to the nine groups of selected publications [C1–C9], in addition to [A, B].

[7] I took Stanley's first QFT two-semester course at Berkeley, Fall 1963 and Spring 1964. There was no textbook. He taught completely from memory and wrote on the blackboard. So it was a special event when one day he brought with him a little strip of paper to class, which had the Klein-Nishina formula on it. The course was unusual and impressive. I had no difficulty writing down notes and studing them. Regrettably, he did not teach his path-field formulations, nor do the current QFT text books! For the anecdote of how Stanley and I coauthored a paper, see the reference at [C5]

[8] The term "string theory" is used to include "bosonic string theory" and "superstring theory," and when specification is needed one of the two latter terms is used.

[9] In addition to being honored by being elected a Fellow of the Royal Society, 1962, Stanley received the Dirac Medal and Prize from the International Centre for Theoretical Physics, 1991; became a Fellow of the American Academy of Arts and Sciences (AAAS), 1992; and received the Dannie Heineman Prize for Mathematical Physics, APS, 1992.

[10] See the web page of the Department of Physics, UC Berkeley, http://physics.berkeley.edu/remembering-stanley-mandelstam; the author's Physics Today Obituary for Stanley Mandelstam, May issue 2017; and this *Memorial Volume*. Also see the extensive biographic writings about Stanley in the *Memorial Volume*: S. Lee, "Stanley Mandelstam: The early years at a "Most Stimulating Theoretical Group," in Birmingham 1954–57; C. Thorn, "Scientific Biography of Stanley Mandelstam, Part I: 1955–1980", and N. Berkovits, "Scientific Biography of Stanley Mandelstam:1981–2016." Thorn's paper in the *Memorial Volume* also gives an almost-exhaustive list of Stanley's publications. Hopefully, by including the several additions selected in this paper, his list will be the exactly-exhaustive list of Stanley's publications.

(S1) The Mandelstam representations in the Mandelstam variables for S-matrices [C1]: The Mandelstam representations are double dispersion relations of S-matrices. The Mandelstam variables are the Lorentz-transformation invariant variables of all possible combinations of the $(3+1)$-dimension momenta of particles involved in the scattering. They form the fundamental framework and strategy for studying S-matrices. They in turn can help the development of quantum theories in which scattering amplitudes can be calculated.[11]

(S2) The S-matrix approach [C2]: In the 1959–61 papers, working with Chew,[12] and then Chew and Frautschi,[13] using the Mandelstam representations and the Regge pole[14] idea, he helped to develop the S-matrix approach for strong interactions. So what Chew had developed with his earlier collaborators: giving the interpretation of particles being poles in the energy-complex-plane and the idea of bootstrap (or nuclear democracy, or duality, or crossing symmetry) got important extensions. Chew advocated it as an alternative to QFT,[15] commonly called the S-matrix theory.[16]

Now QCD (quantum chromodynamics), a QFT, has been established as the theory for strong interactions. Due to its asymptotic freedom, the coupling strength is small at high energies, so perturbative calculations apply. QCD in high energies has been actively researched using both the perturbative QFT and the S-Matrix approaches, as Stanley had always practiced since the beginning of his career.

(S3) The Mandestam path-field formulation for quantum gauge theories and Feynman rules [C3]: His 1962 paper of [C3] used gauge-invariant path-dependent fields for quantum electrodynamics (QED, Maxwell gauge fields interacting with matter fields). Stanley was the first to give such a formulation after the 1959 Aharonov–Bohm (or AB) paper[17] showing the AB effects. Stanley

[11] That was what Stanley did in his 1959 paper with the title, "Construction of the Perturbation Series for Transition Amplitudes from their Analyticity and Unitarity Properties," selected in [C1]. This strategy is still being popularly used.
[12] G. F. Chew, "Recollections of Stanley Mandelstam," a contribution to the *Memorial Volume*.
[13] S. Frautschi, "Stanley Mandelstam and my postdoctoral years at Berkeley," a contribution to the *Memorial Volume*. The paper gave a truly moving story about Stanley, and credited Stanley for introducing the Regge pole idea for obtaining high energy behaviors of S-matrices, which even led to Frautschi's being hired by Caltech because of Frautschi's good work on it.
[14] T. Regge, "Introduction to Complex Orbital Momenta," *Nuovo Cim.* **14**, 951 (1959); "Bound States, Shadow States and Mandelstam Representation," *Nuovo Cim.* **18**, 948 (1960).
[15] G. F. Chew, "*The Analytic S Matrix*," (Benjamin, New York, 1966).
[16] Here the term "S-matrix theory" is used by the author to mean a stand-alone theory; otherwise the term "S-matrix approach" is used, because S-matrices can be and ought to be studied in any theory for particles.
[17] Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in quantum theory," *Phys. Rev.* **115**, 485–491 (1959). Stanley did not refer to this AB paper in the References of his 1962 paper in [C3], but in the text he emphasized the important implications of the AB effects.

considered the AB effect a call from nature. His formulation gave the precise answer. Then in the 1968 paper of [C3], he extended the formulation to include the Yang–Mills gauge fields interacting with matter fields, and in addition developed Feynman rules for perturbative calculations. We call this formulation the Mandelstam path-field formulation for quantum gauge theories.

Now experiments have established that the Standard Model of particle physics are QFTs of Yang–Mills fields interacting with matter fields: QCD for strong interactions and EWT (electroweak theory) for electroweak interactions. The Mandelstam path-field formulation has been actively used.[18] In addition, it has been cited in lattice gauge theory research,[19] as well as in loop quantum gravity research.[20]

In 1974, C. N. Yang[21] gave what he called the integral formalism for gauge theories (Maxwell and Yang–Mills). It involves path-dependent integrations. He discussed its advantages, gave geometric understandings to it, and made contrast to the differential formalism in which he and Mills constructed the Yang–Mills theory.[22] Using his integral formalism, Yang constructed a new theory for gravity.[23] In 1975, T. T. Wu and C. N. Yang[24] made the connection of the integral formalism to the mathematics of fiber bundles. The two papers received large number of citations. Yang's integral formalism has deepened and widened the mathematical-physics perspective for physics and for the Mandelstam path-field formulation.

In 1974 K. Wilson[25] gave the formulation of the Lattice gauge theory for computation. It naturally embodied the ideas of Mandelstam and Yang for continuous gauge theories: being an integral formulation that is path-dependent and gauge-invariant. The lattice gauge theory computations have been hugely successful in producing results that agree with experiments.[26]

[18] J. Terning, "Gauging non-local Lagrangians," *Phys. Rev. D* **44**, 887–897 (1991); C. Csaki, C. Grojean and J. Terning, "Alternatives to an elementary Higgs," *Rev. Mod. Phys.* **88**, 045001 (2016).
[19] M. Creutz, "Gauge fixing, the transfer matrix, and confinement on a lattice," *Phys. Rev. D* **15**, 1128–1136 (1977).
[20] C. Rowelli, "Ashtekar formulation of general relativity and loop-space non-perturbative quantum gravity: A report," *Class. Quant. Grav.* **8**, 1613–1676 (1991).
[21] C. N. Yang, "Integral formalism for gauge fields," *Phys. Rev. Lett.* **33**, 445–447 (1974) [Erratum: *ibid.* **35**, 1748 (1975).
[22] C. N. Yang and R. Mills, "Isotopic spin conservation and a generalized gauge invariance," *Phys. Rev.* **95**, 631–631 (1954); "Conservation of isotopic spin and isotopic gauge invariance," *Phys. Rev.* **96**, 191–195 (1954).
[23] See a more extensive discussion by C. N. Yang, "Gauge fields," pp. 487–561, in the *Proceedings of the Sixth Hawaii Topical Conference in Particle Physics*, Honolulu (University of Hawaii Press, 1976).
[24] T. T. Wu and C. N. Yang, "Concept of nonintegrable phase factors and global formulation of gauge fields," *Phys. Rev. D* **12**, 3845–3857 (1975).
[25] K. Wilson, "Confinement of quarks," *Phys. Rev. D* **10**, 2445 (1974).
[26] M. Creutz, "The lattice and quantized Yang–Mills theory," pp. 41–52 in the *Proceedings of Conference on 60 Years of the Yang–Mills Theory* (World Scientific, May 2016).

Now the terms "Wilson-loop" fields/formulations are almost universally used even when discussing continuous (non-lattice) gauge theories. The term "loop" is often misused, because the paths in the path-dependent gauge field associated with matter fields are open paths, not loops (closed paths), as made clear in Stanley's 1962 and 1968 papers of [C3].

(S4) The Mandelstam path-field formulation for quantum general relativity and the Feynman rules [C4]: In the same year as he pioneered his path-field formulation for quantum gauge theories, Stanley impressively also pioneered his path-field formulation for quantum general relativity, using coordinate-independent (diffeomorphism-invariant) path-dependent fields. These are Stanley's precise response to the call from the basic principle of Einstein's derivation of his general relativity theory.[27] Bryce S. DeWitt, in his highly cited 1967 paper,[28] referred to this 1962 Mandelstam paper of [C4] to be "*the most beautiful attempt at such a language*." However, DeWitt deemed the Mandelstam path-field formulation for quantum general relativity impractical. Nevertheless Stanley pushed on. Later in his 1968 paper of [C4], which referenced this 1967 paper by DeWitt, he derived the Feynman rules in his path-field formulation for quantum general relativity.

The ultraviolet divergences in perturbative QFT for general relativity called for further developments. One being actively pursued, as a competitor to superstring theory, is the loop quantum gravity approach,[29] which embodies the idea of using coordinate-independent (diffeomorphism-invariant) and path-dependent fields, as Mandelstam had practiced in his 1962 paper of [C4].

The idea of using path-fields, instead of local-fields, has also recently been applied in superstring theory researches,[30] through the AdS/CFT (gravity/conformal-field-theory) correspondence.[31]

(S5) The precursor for the discovery of string theory [C5]: [32] In the 1962–1963 papers [C5], two from several, Stanley published the results of his extensive

[27] The manifestation of Mandelstam's coordinate-independent (diffeomorphism-invariant) path-dependent fields in Aharonov–Bohm-type experiments have recently been discussed in the literature.

[28] B. S. DeWitt, "Quantum theory of gravity. II. The manifestly covariant theory," *Phys. Rev.* **162**, 1195–1239 (1967).

[29] See the review paper on loop quantum gravity by C. Rovelli given in an earlier reference in the footnote format.

[30] B. Czech, L. Lamprou, S. McCandlish, S. B. Mosk and J. Sully, "A stereoscopic look into the bulk," *J. High Energy Phys.* **1607**, 129 (2016); 1604.03110, and references therein.

[31] I. R. Klebanov and J. M. Maldacena, "Solving quantum field theories via curved spacetimes," *Physics Today*, January 2009, pp. 28–33.

[32] See the moving tribute to Stanley, with expertise details, by P. Goddard, "The guiding influence of Stanley Mandelstam, from S-matrix theory to string theory," a contribution to the *Memorial Volume*.

work to extend the Regge-pole idea to the general analyticity properties of the complex-plane of angular momentum. In his 1967 and 1968 papers of [C5], using the approximation of straight-line Regge trajectories, including taking the limit to infinity, he gave a theoretical model of S-matrices, later called the dual-resonance model. Amazingly, about a year later (according to the submissions dates of the papers), the Veneziano amplitude[33] became an explicit example of the model, and triggered the eventual discovery of string theory![34]

(S6) The elucidation of mechanisms for quark confinement in QCD [C6]: He took a leading role in elucidating the non-perturbative mechanisms for quark confinement and the phases in QCD. See the many publications selected in [C6]. These are highly influential and quoted papers, considering QCD now is the theory for strong interactions.

(S7) Non-perturbative constructions of the bosonization (or fermionization) in $(1+1)$-dimension QFTs [C7]: Stanley showed non-perturbatively by explicit construction, inspired by the perturbative results of S. Coleman, that operators for the creation and annihilation of quantum sine-Gordon solitons satisfy the anticommutation relations and field equations of the massive Thirring model. Thus he pioneered the non-perturbative operator transformation, now called bosonization or fermionization, that relate a $(1+1)$-dimension bosonic field theory to a $(1+1)$-dimension fermionic field theory. These papers are highly influential in condensed matter and mathematical physics.

In 1984, E. Witten generalized the nonperturbative construction to non-Abelian bosonization in $(1+1)$-dimension and showed that any fermi theory in $(1+1)$-dimension is equivalent to a local bose theory which manifestly possesses all the symmetries of the fermi theory.[35] This paper has revealed a new horizon for mathematical physics and for string theory.

(S8) The proof of perturbative ultraviolet finiteness and $\beta = 0$, in any gauge to all orders, of $N = 4$ SYM [C8]: Stanley alone, as was the case through most of his life, reached the finishing line first in the distribution and submission for publication of the paper that gave the proof in August 1982.

About two months later, in November 1982, the paper of L. Brink, O. Lindgren and B. E. W. Nilsson, who had been independently pursuing the same problem, was submitted for publication.[36] It reached the same result. In this paper they referenced

[33] G. Veneziano, "Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories," *Nuovo Cim. A* **57**, 190–7. [ricevuto (received) il 29 Luglio (July) 1968, while the 1968 [C5] paper of Mandelstam was received 5 September 1967].

[34] David Gross called Stanley "The Godfather of String Theory" when he paid tribute to Stanley as the concluding speaker at Strings 2016, Beijing, China, August 1–5, 2016.

[35] E. Witten, "Non-Abelian bosonization in two dimensions," *Comm. Math. Phys.* **92**, 455–472 (1984).

[36] L. Brink, O. Lindgren, and B. E. W. Nilsson, "The ultraviolet finiteness of the $N = 4$ Yang–Mills theory," *Phys. Lett. B* **123**, 323–328 (1983).

Mandelstam's preprint: Berkeley preprint, UCB-PTH-82/15 (August 1982), and commented on the differences in their perturbative approaches.[37]

Later it was reasoned that this result of ultraviolet finiteness and $\beta = 0$, in any gauge to to all oders, for $N = 4$ SYM holds even in the presence of the nonperturbative effects of instantons, by T. Banks and N. Seiberg in 1986 and again by N. Seiberg in 1988.[38]

Being a QFT from superstring theory in some limit and being a popular model for studying the AdS/CFT correspondences from superstring theory, $N = 4$ SYM has been actively studied for learning quantum gravity that is on the other side of the AdS/CFT correspondence. The result of its being ultraviolet finite has helped to give guidance and checks for other studies, e.g. calculating the S-matrices.

Interestingly, there is the Leibbrandt–Mandelstam prescription, which was developed out of the methods given in Stanley's 1983 paper of [C8], used in the computation of quark and gluon distribution functions for very large nuclei![39]

These accomplishments, commented in (S1–S8) coresponding to the selected papers [C5–C9]), have established Stanley as a supreme creative master of QFT (perturbative as well as non-perturbative) and of the S-matrix approach.

(S9) Many important publications on string theory and eventually the first long-waited proof of pertubative ultraviolet finiteness of superstring theory so it can be considered as a contender for being the theory of quantum gravity [C9]: Since 1986 Stanley had made many important contributions to the development of string theory.[40] Ultimately he derived explicit formulas for all n-loop superstring amplitudes and showed their ultraviolet finiteness and the absence of ambiguities in his 1992 paper of [C9]! Stanley worked years toward finding out the result, and produced Ph.D. students working on the subject. In his 1992 paper of [C9], he acknowledged three for helpful discussions (in alphabetical order with others): A. Berera, N. Berkovits, and S. J. Sin.[41] He also referred to two papers by Berkovits. Later Berkovits showed the perturbative ultraviolet finiteness of superstring theory in different ways.[42]

[37] See also, L. Brink, "Stanley Mandelstam and me and life on the light-cone," a contribution to the *Memorial Volume*.

[38] T. Banks and N. Seiberg, "Non-perturbative infinities," *Nucl. Phys. B* **273**, 157–164 (1986); N. Seiberg, "Supersymmetry and nonperturbative beta functions," *Phys. Lett. B* **206**, 75–80 (1988).

[39] L. McLerran and R. Venugopalan, "Computing quark and gluon distribution functions for very large nuclei," *Phys. Rev. D* **49**, 2233–2241 (1994).

[40] J. Schwarz, "Reminiscences of Stanley Mandelstam," J. Polchinski (Ph.D. 1980, advisor Stanley), "Grad school with Stanley Mandelstam", and papers by Thorn and by Berkovits referenced before, all contributed to the *Memorial Volume*.

[41] Each of all three, A. Berera (1992 Ph.D.), N. Berkovits (1988 Ph.D.) and S. J. Sin (1989 Ph.D.), have contributed a paper to the *Memorial Volume*.

[42] N. Berkovits, "Finiteness and unitarity of Lorentz-covariant Green–Schwarz superstring amplitudes," *Nucl. Phys. B* **408**, 43–61 (1993); "Multiloop amplitudes and vanishing theorems using the pure spinor formalism for the superstring," *JHEP* **0409**, 047 (2004); See also Berkovits' paper contributed to the *Memorial Volume* referenced before.

So, Stanley was also a supreme master of string theory according his work described in (S5,S9) coresponding to the selected papers [C5,C9]).

3. A Perspective

Interestingly, despite its origin being motivated by the studies of strong interactions,[43] superstring theory now is mainly studied as a framework for quantum gravity because of its perturbative ultraviolet finiteness. [44] The experimentally established Standard Model of particle physics, which includes EWT for electroweak interactions[45] and QCD for strong interactions[46] are all regular QFTs. It has no need of the two important necessary ingredients for superstring theory: supersymmetry and more-than-$(3+1)$ extra-dimensions. The Standard Model is not yet embraced in the superstring theory framework,[47] nor have the supersymmetry and extra-dimensions yet been seen experimentally. Superstring theory advocates very much hope and strive for that some limit and/or duality of superstring theory might lead to the inclusion of the Standard Model, as well as to the discovery of the ultimate unification of all interactions, described in one mathematical framework. To all these and more, Stanley had made important contributions, as elaborated in (S1–S9) of the previous section.[48]

Stanley was a supreme master of QFT, the S-matrix approach, and string theory. His influences on theoretical and mathematical physics are deep and diverse, in almost all the current major research efforts trying to deepen our understanding of the physical universe. Stanley's influences live on.

4. Selected Publications

The selections are organized according to subjects given in the previous section and in chronological order of the first publication in each subject, except [C9] which has the latest selected publication of 1992.

[A] **W. Yourgrau and S. Mandelstam**
"*Variational Principles in Dynamics and Quantum Theory*," 1st edition (1952); 2nd edition (1961); 3rd edition, (Dover Publications Inc. NY 1968).[49]

[43] To them Stanley made important contributions (S1, S2, S5).
[44] That was proven first by Stanley in 1992 (S9).
[45] To them Stanley's path-field formulation has been applied for more efficient calculations (S3).
[46] Its quark confinement mechanisms Stanley had elucidated (S6).
[47] This was very much in Stanley's mind, see his 1985 paper in [C9].
[48] "***Stanley Mandelstam's nine dragons in theoretical physics***," we call his nine subjects of achievements.
[49] Wolfgang Yourgrau obtained Ph.D. in physics from Humboldt University, Berlin, Germany in 1932 (while in Berlin he had studied under A. Einstein) and was forced by the Nazi's to "wander" around the world and happened to overlap with Stanley's study at Wits in South Africa!
The 3rd edition has the prefaces of earlier editions. They gave acknowledgements to Werner Heisenberg, Erwin Schwinger, L. de Broglie, M. Born, etc.! Also the authors' institutions and titles in the prefaces gave information of their careers 1952-1968. The third edition was published in 1968 after Stanley had settled down for good as a professor in Physics at UC Berkeley and Wolfgand as a professor in History and Philosophy of Science at the University of Denver, coincidently both in 1963.

[B] **S. Mandelstam**

"Dynamical variables in the Bethe-Salpeter formalism," *Proceedings of the Royal Society of London A* **233**, 246–266 (1955);

"Uniqueness of solutions of the Bethe-Salpeter equation for scattering," *ibid.* **237**, 496–516 (1956). [50]

[C1] **S. Mandelstam**

"Determination of the pion-nucleon scattering amplitude from dispersion relations and unitarity: general theory," *Phys. Rev.* **112**, 1344–1360 (1958);

"Analytic properties of transition amplitudes in perturbation theory," *Phys. Rev.* **115**, 1741–1751 (1959);

"Construction of the perturbation series for transition amplitudes from their analyticity and unitarity properties," *Phys. Rev.* **115**, 1752 (1959);

"Unitarity condition below physical thresholds in the normal and anomalous cases," *Phys. Rev. Lett.* **4**, 84 (1960);

"Some rigorous properties of transition amplitudes," *Nuovo Cim.* **15**, 658 (1960);

"Two-dimensional representations of scattering amplitudes and their applications," pp. 209–233, in "*Quantum Theory of Fields*," *Proceedings of the 1961 Twelfth Solvay Conference on Physics*, Chair: Sir Lawrence Bragg (Cambridge), October, 1961, University of Brussels, Belgium, Eds. R. Stoops, (Interscience Publishers, a division of John Wiley & Sons, Inc., New York, 1961).

[C2] **G. F. Chew and S. Mandelstam**

"Theory of low-energy pion pion interactions," *Phys. Rev.* **119**, 467–477 (1960);

"Theory of low-energy pion pion interactions II," *Nuovo Cim.* **19**, 752 (1961);

S. Mandelstam

"Dispersion relations in strong-coupling physics," in *Reports on Progress in Physics XXV*, pp. 99–162 (1962), ed. A. C. Stickland; an extensive review referencing earlier papers (up to and without the next paper) and Regge paper;

G. F. Chew, S. C. Frautschi and S. Mandelstam

"Regge poles in pi pi scattering," *Phys. Rev.* **126**, 1202–1208 (1962);

S. Mandelstam, J. E. Paton, Ronald F. Peierls, and A. Q. Sarker.[51]
"Isobar approximation of production processes," *Annals Phys.* **18**, 198–225 (1962).

[50] Stanley's Ph.D. thesis publications, both communicated by Rudolf Peierls, his thesis advisor.
[51] Ronald and I coauthored papers. The one with T. L. Trueman we are most proud of, "Estimates of production cross sections and distributions for W bosons and hadrons jets in high energy pp and $p\bar{p}$ collisions," *Phys. Rev. D* **16**, 1397 (1977). Our results for the quantities in the title agreed with the 1983 experiments, whose observation of W^+, W^- and Z^0 earned the 1984 Nobel Prize in Physics, http://www.nobelprize.org/nobel_prizes/physics/laureates/1984/.

[C3] **S. Mandelstam**

"Quantum electrodynamics without potentials," *Annals Phys.* **19**, 1–24 (1962);

"Feynman rules for electromagnetic and Yang–Mills fields from the gauge independent field theoretic formalism," *Phys. Rev.* **175**, 1580–1623 (1968).

[C4] **S. Mandelstam**

"Quantization of the gravitational field," *Annals Phys.* **19**, 25–66 (1962);

"Feynman rules for the gravitational field from the coordinate independent field theoretic formalism," *Phys. Rev.* **175**, 1604–1623 (1968).

[C5] **S. Mandelstam**

"An extension of the Regge formula," *Annals Phys.* **19**, 254–261 (1962);

"Regge poles as consequences of analyticity and unitarity," *Annals Phys.* **21**, 302–343 (1963);

S. Mandelstam and L. L. Wang (now Chau)[52]

"Gribov-Pomeranchuk Poles in Scattering Amylitudes," *Phys. Rev.* **160**, 1490–1493 (1967);

S. Mandelstam

"Dynamics based on indefinitely rising Regge trajectories," pp. 604–615, in the *Proceedings of the 1967 Rochester International Conference on Particles and Fields*, University of Rochester, Rochester, New York, August 28–September 1, 1967, eds. R. Hagen, G. Guralnik and V. S. Mathur (Interscience Publishers, 1967);

"Dynamics based on rising Regge trajectories," *Phys. Rev.* **166**, 1539–1552 (1968). As mentioned on the page of (S5), this paper was received on September 5, 1967, while the 1968 Veneziano-amplitude paper, *Nuovo Cim. A* **57**, 190–7 (1968), was received on July 29, 1968, and did refer to this paper of Mandelstam in Ref. 2.

[52] I included this paper here so I can tell the story how it came about. I got my Ph.D. in 1966 under the guidance of Geoffrey Chew. My thesis was essentially a clipping together of three published papers with a covering page. For family reasons, I delayed my graduation and then stayed on working at Berkeley until Fall 1967 (before I went to Institute for Advanced Study at Princeton). During that time, one day I saw the announcement of a talk to be given by Stanley with a title and abstract that seemed like something that I had been working on. I went to the talk, and indeed it was. After the talk, I went to his office and told him that. Further I told him some results that he did not cover in his talk. He was surprised, and he walked to the window and thought. After a few minutes, he came back from the window and said, "You are right. I did not know that." A few days later, he looked me up in my office and handed me a manuscript and said, "Your name should be on this paper." I reviewed it and agreed with the content, which had also included the part that he said he did not know. So I happily agreed to have my name on it and suggested a few minor revisions. That was how our co-authoered paper came about. So we did not really collaborated on that paper. It is interesting to note that it was received by Physical Review on March 20, 1967. Now looking back on his publications, he had very few co-authored papers, totaly only seven: one with Yourgrau at Wit in South Africa, one with Ronald F. Peierls and collaboratories at University of Birmingham, four with Chew and collaborators at Berkeley, and the one with me!

[C6] **S. Mandelstam**

"Vortices and quark confinement in non-Abelian gauge theories," *Phys. Lett. B* **53**, 476–478 (1975);

"Vortices and quark confinement in non-Abelian gauge theories," in pp. 245–249 of *Phys. Rept.* Vol. 23, Issue 3 (1976), which is the *Proceeding of the Meeting on "Extended systems in field theory,"* Ecole Normale Supérieure, Paris, June 16–21, 1975, eds. J. L. Gervais and A. Neveu;

"Charge-monopole duality and the phases of non-Abelian gauge theories," *Phys. Rev. D* **19**, 2391–2401 (1979);

"Approximation scheme for QCD," *Phys. Rev. D* **20**, 3223 (1979);

"General introduction to confinement," pp. 109–121, *Phys. Rept.* Vol. 67, Issue 1 (1980), which is the *Proceedings of Les Houches Winter Advanced Study Institute on "Common trends in particle and condensed matter physics,"* February 1980, eds: E. Brezin, J.-L. Gervais and G. Toulouse.

[C7] **S. Mandelstam**

"Soliton operators for the quantized sine–Gordon equation," *Phys. Rev. D* **11**, 3026–3030 (1975);

"Soliton operators for the quantized sine–Gordon equation," pp. 307-313 of *Phys. Rept.* Vol. 23, Issue 3 (1976), which is the *Proceeding of the Meeting on Extended Systems in Field Theory*, Ecole Normale Supérieure, Paris, June 16–21, 1975, eds. J. L. Gervais and A. Neveu.

[C8] **S. Mandelstam**

"Light cone superspace and the ultraviolet finiteness of the $N = 4$ model," *Nucl. Phys. B* **213**, 149–168 (1983);

"Ultraviolet finiteness of the $N = 4$ model," pp. 167–177, in the book "*High-Energy Physics*," of the series Studies in the Natural Sciences, Volume 20, Authors: Behram Kursunoglu, Editors: Stephan L. Mintz, Arnold Perlmutter, (Springer 1985); ISBN: 978-1-4684-8850-0; In Honor of P. A. M. Dirac in his 80th Year, 17–22 Jan 1983. Miami, Florida.

[C9] **S. Mandelstam**

"Veneziano formula with trajectories spaced by two units," *Phys. Rev. Lett.* **21**, 1724 (1968);

"Generalizations of the Veneziano and Virasoro models," *Phys. Rev.* **183**, 1374 (1969);

"Manifestly dual formulation of the Ramond model," *Phys. Lett. B* **46**, 447 (1973);

"Interacting string picture of dual resonance models," *Nucl. Phys. B* **64**, 205 (1973);

"Interacting string picture of the Neveu–Schwarz–Ramond model," *Nucl. Phys. B* **69**, 77 (1974);

"Dual-resonance models," *Phys. Rept.* **13**, 259–353 (1974);

"Dual-resonance models," pp. 593–637, in *Les Houches, June Institute on "Structural Analysis of Collision Amplitudes,"* June 2–27, 1975, Amsterdam, (North-Holland Pub. Co., New York: American Elsevier Pub. Co., 1976), eds. Roger Balian and Daniel Iagolnitzer;

"Introduction to Strings Model and Vertex Operators," pp. 15–35, in *Proceedings on Vertex Operators in Mathematics and Physics*, eds. P. J. Lepowsky, S. Mandelstam and I. Singer, Berkeley, CA, USA, 1983 (Springer-Verlag, 1985);[53]

"Composite Vector Mesons and String Models," pp. 97–105, in "*A Passion for Physics, Essays in Honor of Geoffrey Chew*," for his sixtieth anniversary, Berkeley, 5 June 1984, eds. C. DeTar, J. Finkelstein, and C. I. Tan (World Scientic, Singapore, 1985);[54]

"Interacting string picture of the fermionic string," *Prog. Theor. Phys. Suppl.* **86**, 163 (1986);

"The n loop string amplitude: Explicit formulas, finiteness and absence of ambiguities," *Phys. Lett. B* **277**, 82–88 (1992).

Acknowledgments

I would like thank Professor Kok Khoo Phua and Professor Lars Brink for inviting me to participate in organizing the "*Memorial Volume for Stanley Mandelstam*" (the *Memorial Volume*) and to contribute a paper. My original plan was to write an article with the title "Endearing Memories of Stanley Mandelstam" about anecdotes and photos of Stanley to show his humor and good-heartedness, and to let the scientific side of Stanley be written up by those contributors to the *Memorial Volume* who are experts/practitioners of string theory — the theory to which Stanley laid the precursor in the 1960s and made concerted efforts to contribute till the end of his life.[55]

Then came the invitation from Physics Today (represented by Ms. Gayle Parraway), asking me to write an Obituary for Stanley Mandelstam (the Obituary). It was a surprise to me. After much hesitation and thought, I decided to take the

[53] I was at the wonderful conference and presented the paper " Supersymmetric Yang-Mills Fields as An Integrable System and Connections with Other Non-linear Systems."

[54] I was at the wonderful celebration and presented the paper "Comments on Heavy Quark Decays and CP Violation".

[55] I have never being a practitioner of string theory, however chance has made me encounter string theory ever since it was still in its 1968 embryonic stage. [That episode was documented in the book *A Brief History of String Theory, from Dual Models to M-Theory* (Springer, Heidelberg, 2014) by D. Rickles. My name was Ling-Lie Wang, as was correctly written, but I was at Institute for Advanced Study, Princeton, not Princeton University as was incorrectly witten.] I wrote about this on the occasion of celebrating John Schwarz's 75th birthday in 2016, with the title "My Encounters with String Theory," for chatting with friends. Though not a practitioner, I have tried to be an alert observer on its development and owned almost all string theory books and even studied some of them.

challenge. While researching and organizing thoughts for writing the Obituary, to my amazement, I discovered many new insights about Stanley's accomplishments and long lasting influences through his publications. Their depth and immensity are truly impressive. I deeply regret that I did not fully appreciated what he had accomplished when he was still living. Telling the story properly compelled me to write this paper. The Obituary has a limit of less than 850 words, in which I could give only the highlights but could not offer any details supported by Stanley's publications or references to other literature. To my pleasant surprise, it has turned out to be beneficial to work both of these two manuscripts. The contributed papers to the *Memorial Volume* have been very helpful to me in writing this paper. Also I am happy to have become better at contributing to the *Memorial Volume* because of the research and thoughts devoted for writing this paper. In our dedication to celebrate Stanley's life for the good of our physics community, my hope is that publishing this paper in the *Memorial Volume* as well as the Obituary in *Physics Today* will serve readers better with their differing focus and depth. I owe sincere thanks to Physics Today for inviting me to write an Obituary for Stanley Mandelstam.

I would also like to give my heartfelt gratitude to the Department of Physics of UC Berkeley, in particular the Director of the Berkeley Center for Theoretical Physics (BCTP) Professor Yasunori Normura, Managers Ms. Eleanor Crump and Mr. Brian Underwood, for their most helpful assistance in providing information and records related to Stanley: photos, especially the photo used as the cover of the *Memorial Volume*, the list of Stanley's Ph.D. students, the exact catalog of Stanley's collection of 232 physics and mathematics books, and requesting fee-waivers for reprinting some of Stanley's papers in this *Memorial Volume*. I am touched by their gracious offer to me to have all Stanly's books! I gratefully accepted the three that Stanley co-authored and the one that has Stanley's paper. Besides keeping them as mementos, I have put them to good use for writing this paper, as listed in Selected Publications [A, C2, C9].

Now the paper is done, I give my heartfelt thanks to Professors Steven C. Frautschi, Yasunori Nomura, David Pines, John H. Schwarz, Nathan Seiberg, and John Terning for reviewing this manuscript and making helpful and encouraging comments, and especially to Steven for saying "Your mention of my Memorial Volume contribution in the reference at (S2) is accurate and appropriate.", to David for saying "You make a very convincing case for Stanley having played a key role in theoretical physics, from the early days of QFT to string theory," and "You are clearly the right person to write it." and to John H. for saying "Everything you wrote is factually correct."; and to Dr. Richard Breedon, Ms. Eleanor Crump, and Mr. Weiben Wang for very helpful editorial comments.

Of course, any errors in this paper are mine alone.

In looking forward to the final production of the *Memorial Volumel* by the World Scientific, I give my sincere appreciation to Mr. Chee Hok Lim of World Scientific for his always outstdanding performance in the production process.

Attachment: Photos of Stanley with colleagues, students, and friends

Fig. 2. The wall size photo of the Solvay Conference on Physics 1961 (which celebrated the 50^{th} anniversary of the 1911 inaugural one), (The author had it installed on the 4th floor of the Department of Physics, UC Davis. in the late 1980s.)
First row, seated, left to right: S. Tomonaga, W. Heitler, Y. Nambu, N. Bohr, E. Perrin, J. R. Oppenheimer, Sir W. L. Bragg, C. Møller, C. J. Goter, H. Yukawa, R. E. Peierls, H. A. Bethe;
Second row: I. Prigogine, A. Pais, A. Salam, W. Heisenberg, F. J. Dyson, R. D. Feynman, L. Rosenfeld, P. A. M. Dirac, L. van Hove, O. Klein;
Third row: A. S. Whightman, S. Mandelstam, G. F. Chew, M. L. Goldberger, G. C. Wick, M. Gell-Mann, G. Källén, E. Wigner, G. Wentzel, J. Schwinger, M. Cini.

Fig. 3. Stanley Fest celebrating his 80th birthday, KITP, UC Santa Barbara, February 13, 2009.

Fig. 4. After-conference dinner, 13th February 2009.

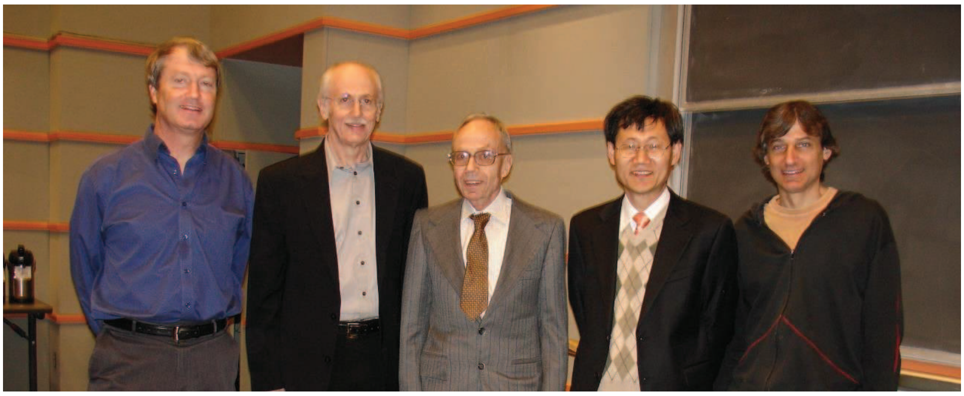

Fig. 5. Stanley (middle) with four of his UC Berkeley Ph.D. students, 13$^{\text{th}}$ February 2009, From left to right: J. Polchinski (Ph.D. 1980), C. Thorn (Ph.D. 1971), S. J. Sin (Ph.D. 1989), N. Berkovits (Ph.D. 1988).

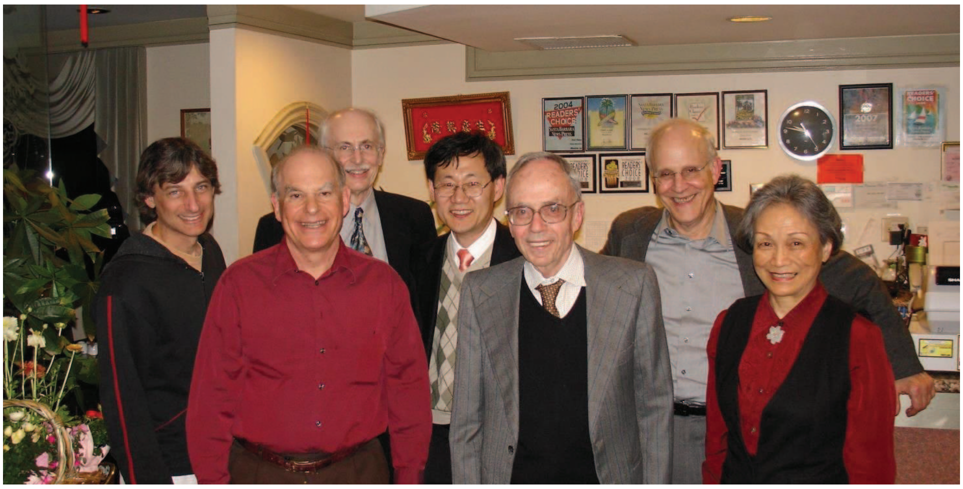

Fig. 6. Stanley (5$^{\text{th}}$ from the left) with six of his UC Berkeley QFT class students, 13$^{\text{th}}$ February 2009, N. Berkovits, J. H. Schwarz, C. Thorn, S. J. Sin, D. J. Gross, L. L. Chau (QFT classes, Fall 1963 & Spring 1964). [Chau, Gross, and Schwarz all got Ph.D. in 1966, with adviser G. F. Chew, at UC Berkeley.]

Fig. 7. S. Mandelstam and G. F. Chew at SFMOMA, San Francisco, 2010, during one of the outings organized by L. L. Chau.

Fig. 8. Annual New Year's Eve Dinner, 31^{st} December 2007, since 1999,
From left to right: G. F. Chew, L. L. Chau, S. Mandelstam,
Sadly now without Stanley from 2016 onward.

Stanley was an exceptional scholarly gentleman (君子). His spirit lives on.